[Mallika, 6(4): April 2019] ISSN 2348 – 8034
DOI- 10.5281/zenodo.2640807 Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

ON BINARY DIOPHANTINE EQUATION

$$8x^2 - 7y^2 = k^2 + 14k - 7$$

S. Mallika[*1] & K. Ramya[2]

[*1]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-2, Tamilnadu, India

[2]M.phil scholar, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-2, Tamilnadu, India

ABSTRACT

Non-homogeneous binary quadratic equation representing hyperbola given by $8x^2 - 7y^2 = k^2 + 14k - 7$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

Keywords: *Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and $N.$ In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $8x^2 - 7y^2 = k^2 + 14k - 7$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non- zero distinct integral solution is

$$8x^2 - 7y^2 = k^2 + 14k - 7 \tag{1}$$

Introduce the linear transformation

$$x = X + 7T, y = X + 8T \tag{2}$$

From (1) & (2) we have

$$X^2 = 56T^2 + k^2 + 14k - 7 \tag{3}$$

whose smallest positive integer solution is

$X_0 = k + 7, T_0 = 1$

To obtain the other solutions of (3), consider the pell equation,

$$X^2 = 56T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

$\tilde{X}_0 = 15, \tilde{T}_0 = 2$

The general solution of (4) is given by,

$$\tilde{X}_n = \frac{1}{2} f_n, \tilde{T}_n = \frac{1}{4\sqrt{14}} g_n$$

where

$$f_n = [(15 + 4\sqrt{14})^{n+1} + (15 - 4\sqrt{14})^{n+1}]$$

$$g_n = [(15 + 4\sqrt{14})^{n+1} - (15 - 4\sqrt{14})^{n+1}]$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solutions of (1) are given by,

$$4\sqrt{14} x_{n+1} = (2k + 28)\sqrt{14} f_n + (7k + 105) g_n$$

$$4\sqrt{14} y_{n+1} = (2k + 30)\sqrt{14} f_n + (8k + 112) g_n$$

The recurrence relations satisfied by x and y are given by,

$$x_{n+1} - 30 x_{n+2} + x_{n+3} = 0$$

$$y_{n+1} - 30 y_{n+2} + y_{n+3} = 0$$

Some numerical examples of x and y satisfying (1) are given in the table 1 below.

Table 1: Numerical examples

n	x_n	y_n
0	$k + 14$	$k + 15$
1	$29k + 420$	$31k + 449$
2	$869k + 12586$	$929k + 13455$
3	$2604k + 377160$	$27839k + 403201$

From the above table, we observe some interesting relations among the solutions which are presented below:

If k is odd, x_n is odd and y_n is even.

If k is even, x_n is even and y_n is odd.

Each of the following expressions is a nasty number.

$$\frac{1}{k^2 + 14k - 7}[(186k + 2694) x_{2n+2} - (6k + 90) x_{2n+3} + 12(k^2 + 14k - 7)]$$

$$\frac{1}{5(k^2 + 14k - 7)}[(929k + 13455) x_{2n+2} - (k + 15) x_{2n+4} + 60(k^2 + 14k - 7)]$$

$$\frac{1}{k^2 + 14k - 7}[(96k + 1344) x_{2n+2} - (84k + 1260) y_{2n+2} + 12(k^2 + 14k - 7)]$$

$$\frac{1}{5(k^2 + 14k - 7)}[(928k + 13440) x_{2n+2} - (28k + 420) y_{2n+3} + 60(k^2 + 14k - 7)]$$

$$\frac{1}{449(k^2+14k-7)}[(83424k+1208256)x_{2n+2}-(84k+1260)y_{2n+4}+5388(k^2+14k-7)]$$

$$\frac{1}{7(k^2+14k-7)}[(39018k+565110)x_{2n+3}-(1302k+18858)x_{2n+4}+84(k^2+14k-7)]$$

$$\frac{1}{5(k^2+14k-7)}[(32k+448)x_{2n+3}-(868k+12572)y_{2n+2}+60(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(2784k+40320)x_{2n+3}-(2604k+37716)y_{2n+3}+12(k^2+14k-7)]$$

$$\frac{1}{5(k^2+14k-7)}[(27808k+402752)x_{2n+3}-(868k+12572)y_{2n+4}+60(k^2+14k-7)]$$

$$\frac{1}{449(k^2+14k-7)}[(96k+1344)x_{2n+4}-(78036k+1130220)y_{2n+2}+5388(k^2+14k-7)]$$

$$\frac{1}{5(k^2+14k-7)}[(928k+13440)x_{2n+4}-(26012k+376740)y_{2n+3}+60(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(83424k+805504)x_{2n+4}-(52024k+753480)y_{2n+4}+12(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(6k+84)y_{2n+3}-(174k+2520)y_{2n+2}+12(k^2+14k-7)]$$

$$\frac{1}{5(k^2+14k-7)}[(k+14)y_{2n+4}-(869k+12586)y_{2n+2}+60(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(174k+2520)y_{2n+4}-(5214k+75516)y_{2n+3}+12(k^2+14k-7)]$$

Each of the following expressions is a cubical integer:

$$\frac{1}{k^2+14k+7}[(31k+449)x_{3n+3}-(k+15)x_{3n+4}+(93k+1347)x_{n+1}-(3k+45)x_{n+2}]$$

$$\frac{1}{30(k^2+14k+7)}[(929k+13455)x_{3n+3}-(k+15)x_{3n+5}+(2787k+40365)x_{n+1}$$
$$-(3k+45)x_{n+3}]$$

$$\frac{1}{k^2+14k+7}[(16k+224)x_{3n+3}-(14k+210)y_{3n+3}+(48k+672)x_{n+1}-(42k+630)y_{n+1}]$$

$$\frac{1}{15(k^2+14k+7)}[(464k+6720)x_{3n+3}-(14k+210)y_{3n+4}+(1392k+20160)x_{n+1}$$
$$-(42k+630)y_{n+2}]$$

$$\frac{1}{449(k^2+14k+7)}[(13904k+201376)x_{3n+3}-(14k+210)y_{3n+5}+(41712k+604128)x_{n+1}$$
$$-(42k+630)y_{n+2}]$$

$$\frac{1}{7(k^2+14k+7)}[(6503k+94185)x_{3n+4}-(217k+3143)x_{3n+5}+(19509k+282555)x_{n+2}$$
$$-(651k+9429)x_{n+3}]$$

$$\frac{1}{15(k^2+14k+7)}[(16k+224)x_{3n+4}-(434k+6286)y_{3n+3}+(48k+672)x_{n+2}$$
$$-(1302k+18858)y_{n+1}]$$

$$\frac{1}{k^2+14k+7}[(464k+6720)x_{3n+4}-(434k+6286)y_{3n+4}+(1392k+20160)x_{n+2}$$
$$-(1302k+18858)y_{n+2}]$$

$$\frac{1}{15(k^2+14k+7)}[(13904k+201376)x_{3n+4}-(434k+6286)y_{3n+5}+(41712k+604128)x_{n+2}$$
$$-(1302k+18858)y_{n+3}]$$

$$\frac{1}{449(k^2+14k+7)}[(16k+224)x_{3n+5}-(13006k+188370)y_{3n+3}+(48k+672)x_{n+3}$$
$$-(39018k+565110)y_{n+1}]$$

$$\frac{1}{15(k^2+14k+7)}[(464k+6720)x_{3n+5}-(13006k+188370)y_{3n+4}+(1392k+20160)x_{n+3}$$
$$-(39018k+565110)y_{n+2}]$$

$$\frac{1}{k^2+14k+7}[(13094k+201376)x_{3n+5}-(13006k+188370)y_{3n+5}+(41712k+604128)x_{n+3}$$
$$-(39018k+565110)y_{n+3}]$$

$$\frac{1}{k^2+14k+7}[(k+14)y_{3n+4}-(29k+420)y_{3n+3}+(3k+42)y_{n+2}-(87k+1260)y_{n+1}]$$

$$\frac{1}{30(k^2+14k+7)}[(k+14)y_{3n+5}-(869k+12586)y_{3n+3}+(3k+42)y_{n+3}$$
$$-(2607k+37758)y_{n+1}]$$

$$\frac{1}{k^2+14k+7}[(29k+420)y_{3n+5}-(869k+12586)y_{3n+4}+(87k+1260)y_{n+3}$$
$$-(2607k+37758)y_{n+2}]$$

[Mallika, 6(4): April 2019] ISSN 2348 – 8034
DOI- 10.5281/zenodo.2640807 Impact Factor- 5.070

Each of the following expressions is a biquadratic integer:

$$\frac{1}{k^2+14k-7}[(31k+449)x_{4n+4}-(k+15)x_{4n+5}+(124k+1796)x_{2n+2}$$
$$-(4k+60)x_{2n+3}+6(k^2+14k-7)]$$

$$\frac{1}{30(k^2+14k-7)}[(929k+13455)x_{4n+4}-(k+15)x_{4n+6}+(3716k+53820)x_{2n+2}$$
$$-(4k+60)x_{2n+4}+180(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(16k+224)x_{4n+4}-(14k+210)y_{4n+4}+(64k+896)x_{2n+2}$$
$$-(56k+840)y_{2n+2}+6(k^2+14k-7)]$$

$$\frac{1}{15(k^2+14k-7)}[(464k+6720)x_{4n+4}-(14k+210)y_{4n+5}+(1856k+26880)x_{2n+2}$$
$$-(56k+840)y_{2n+3}+90(k^2+14k-7)]$$

$$\frac{1}{449(k^2+14k-7)}[(13904k+201376)x_{4n+4}-(14k+210)y_{4n+6}+(55616k+805504)x_{2n+2}$$
$$-(56k+840)y_{2n+4}+2694(k^2+14k-7)]$$

$$\frac{1}{7(k^2+14k-7)}[(6503k+94185)x_{4n+5}-(217k+3143)x_{4n+6}+(26012k+376740)x_{2n+3}$$
$$-(868k+12572)x_{2n+4}+42(k^2+14k-7)]$$

$$\frac{1}{15(k^2+14k-7)}[(16k+224)x_{4n+5}-(434k+6286)y_{4n+4}+(64k+896)x_{2n+3}$$
$$-(1736k+25144)y_{2n+2}+90(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(464k+6720)x_{4n+5}-(434k+6286)y_{4n+5}+(1856k+26880)x_{2n+3}$$
$$-(1736k+25144)y_{2n+3}+6(k^2+14k-7)]$$

$$\frac{1}{15(k^2+14k-7)}[(13904k+201376)x_{4n+5}-(434k+6286)y_{4n+6}+(55616k+805504)x_{2n+3}$$
$$-(1736k+25144)y_{2n+4}+90(k^2+14k-7)]$$

$$\frac{1}{449(k^2+14k-7)}[(16k+224)x_{4n+6}-(13006k+188370)y_{4n+4}+(64k+896)x_{2n+4}$$
$$-(52024k+753480)y_{2n+2}+2694(k^2+14k-7)]$$

$$\frac{1}{15(k^2+14k-7)}[(464k+6720)x_{4n+6}-(13006k+188370)y_{4n+5}+(1856k+26880)x_{2n+4}$$
$$-(52024k+753480)y_{2n+3}+90(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(13904k+201376)x_{4n+6}-(13006k+188370)y_{4n+6}+(55616k+805504)x_{2n+4}$$
$$-(52024k+753480)y_{2n+4}+6(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(k+14)y_{4n+5}-(29k+420)y_{4n+4}+(4k+56)y_{2n+3}$$
$$-(116k+1680)y_{2n+2}+6(k^2+14k-7)]$$

$$\frac{1}{30(k^2+14k-7)}[(k+14)y_{4n+6}-(869k+12586)y_{4n+4}+(4k+56)y_{2n+4}$$
$$-(3476k+50344)y_{2n+2}+180(k^2+14k-7)]$$

$$\frac{1}{k^2+14k-7}[(29k+420)y_{4n+6}-(869k+12586)y_{4n+5}+(116k+1680)y_{2n+4}$$
$$-(3476k+50344)y_{2n+3}+6(k^2+14k-7)]$$

Each of the following expressions is a quintic integer:

$$\frac{1}{(k^2+14k-7)}[(31k+449)x_{5n+5}-(k+15)x_{5n+7}+(155k+2245)x_{3n+3}$$
$$-(5k+75)x_{3n+4}+(310k+4490)x_{n+1}-(10k+150)x_{n+2}]$$

$$\frac{1}{30(k^2+14k-7)}[(929k+13455)x_{5n+5}-(k+15)x_{5n+7}+(4645k+67275)x_{3n+3}$$
$$-(5k+75)x_{3n+5}+(9290k+134550)x_{n+1}-(10k+150)x_{n+3}]$$

$$\frac{1}{(k^2+14k-7)}[(16k+224)x_{5n+5}-(14k+210)y_{5n+5}+(90k+1120)x_{3n+3}$$
$$-(70k+1050)y_{3n+3}+(160k+2240)x_{n+1}-(140k+2100)y_{n+1}]$$

$$\frac{1}{15(k^2+14k-7)}[(464k+6720)x_{5n+5}-(14k+210)y_{5n+6}+(2320k+33600)x_{3n+3}$$
$$-(70k+1050)y_{3n+4}+(4640k+67200)x_{n+1}-(140k+2100)y_{n+2}]$$

$$\frac{1}{449(k^2+14k-7)}[(13904k+201376)x_{5n+5}-(14k+210)y_{5n+7}+(69520k+1006880)x_{3n+3}$$
$$-(70k+1050)y_{3n+5}+(139040k+2013760)x_{n+1}-(140k+2100)y_{n+2}]$$

$$\frac{1}{7(k^2+14k-7)}[(6503k+94185)x_{5n+6}-(217k+3143)x_{5n+7}+(32515k+470925)x_{3n+4}$$
$$-(1085k+15715)x_{3n+5}+(65030k+941850)x_{n+2}-(2170k+31430)x_{n+3}]$$

$$\frac{1}{15(k^2+14k-7)}[(16k+224)x_{5n+6}-(434k+6286)y_{5n+5}+(90k+1120)x_{3n+4}$$
$$-(2170k+31430)y_{3n+3}+(160k+2240)x_{n+2}-(4340k+62860)y_{n+1}]$$

$$\frac{1}{(k^2+14k-7)}[(464k+6720)x_{5n+6}-(434k+6286)y_{5n+6}+(2320k+33600)x_{3n+4}$$
$$-(2170k+31430)y_{3n+4}+(4640k+67200)x_{n+2}-(4340k+62860)y_{n+2}]$$
$$\frac{1}{15(k^2+14k-7)}[(13904k+201376)x_{5n+6}-(434k+6286)y_{5n+7}+(69520k+1006880)x_{3n+4}$$
$$-(2170k+31430)y_{3n+5}+(139040k+2013760)x_{n+2}-(4340k+62860)y_{n+3}]$$
$$\frac{1}{449(k^2+14k-7)}[(16k+224)x_{5n+7}-(13006k+188370)y_{5n+5}+(80k+1120)x_{3n+5}$$
$$-(65030k+941850)y_{3n+3}+(160k+2240)x_{n+3}-(130060k+1883700)y_{n+1}]$$
$$\frac{1}{15(k^2+14k-7)}[(464k+6720)x_{5n+7}-(13006k+188370)y_{5n+6}+(2320k+33600)x_{3n+5}$$
$$-(65030k+941850)y_{3n+4}+(4640k+67200)x_{n+3}-(130060k+183700)y_{n+2}]$$
$$\frac{1}{(k^2+14k-7)}[(13904k+201376)x_{5n+7}-(13006k+188370)y_{5n+7}+(69520k+1006880)x_{3n+5}$$
$$-(65030k+941850)y_{3n+5}+(139040k+2013760)x_{n+3}-(130060k+1883700)y_{n+3}]$$
$$\frac{1}{(k^2+14k-7)}[(k+14)y_{5n+6}-(29k+420)y_{5n+5}+(5k+70)y_{3n+4}$$
$$-(145k+2100)y_{3n+3}+(10k+140)y_{n+2}-(290k+4200)y_{n+1}]$$
$$\frac{1}{30(k^2+14k-7)}[(k+14)y_{5n+7}-(869k+12586)y_{5n+5}+(5k+70)y_{3n+5}$$
$$-(4345k+62930)y_{3n+3}+(10k+140)y_{n+3}-(8690k+125860)y_{n+1}]$$
$$\frac{1}{(k^2+14k-7)}[(29k+420)y_{5n+7}-(869k+12586)y_{5n+6}+(145k+2100)y_{3n+5}$$
$$-(4345k+62930)y_{3n+4}+(290k+4200)y_{n+3}-(8690k+125860)y_{n+2}]$$

Relations satisfied by the solutions are as follows:

$x_{n+3}=30x_{n+2}-x_{n+1}$.

$14y_{n+1}=x_{n+2}-15x_{n+1}$

$14y_{n+2}=15x_{n+2}-x_{n+1}$

$14y_{n+3}=449x_{n+2}-15x_{n+1}$

$420y_{n+1}=x_{n+3}-449x_{n+1}$

$16y_{n+2}=x_{n+3}-x_{n+1}$

$420y_{n+3}=449x_{n+3}-x_{n+1}$

$y_{n+2}=14x_{n+1}+15y_{n+1}$

$y_{n+3}=480x_{n+1}+449y_{n+1}$

RESEARCHERID
THOMSON REUTERS
[Mallika, 6(4): April 2019] ISSN 2348 – 8034
DOI- 10.5281/zenodo.2640807 Impact Factor- 5.070

$$x_{n+3} = 28y_{n+2} + x_{n+1}$$
$$15y_{n+1} = y_{n+2} - 16x_{n+1}$$
$$15y_{n+3} = 16x_{n+1} + 449y_{n+2}$$
$$14y_{n+1} = 15x_{n+3} - 449x_{n+2}$$
$$14y_{n+3} = 15x_{n+3} - x_{n+2}$$
$$14y_{n+2} = x_{n+3} - 15x_{n+2}$$
$$15y_{n+2} = 16x_{n+2} + y_{n+1}$$
$$y_{n+3} = 32x_{n+2} + y_{n+1}$$
$$y_{n+3} = 16x_{n+2} + 15y_{n+2}$$
$$449y_{n+2} = 16x_{n+3} + 15y_{n+1}$$
$$449y_{n+3} = 480x_{n+3} + y_{n+1}$$
$$15y_{n+3} = 16x_{n+3} + y_{n+2}$$
$$y_{n+3} = 30y_{n+2} - y_{n+1}$$

Remarkable Observations:
Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in the table 2 below:

Table 2: Hyperbolas

S.No:	Hyperbola	(X,Y)
1)	$7X^2 - 2Y^2 = 28(k^2 + 14k - 7)^2$	$[(31k+449)x_{n+1} - (k+15)x_{n+2},$ $(2k+28)x_{n+2} - (58k+840)x_{n+1}]$
2)	$56X^2 - Y^2 = 201600(k^2 + 14k - 7)^2$	$[(929k+13455)x_{n+1} - (k+15)x_{n+3},$ $(8k+112)x_{n+3} - (6952k+100688)x_{n+1}]$
3)	$X^2 - 14Y^2 = 4(k^2 + 14k - 7)^2$	$[(16k+224)x_{n+1} - (14k+210)y_{n+1},$ $(4k+56)y_{n+1} - (4k+60)x_{n+1}]$
4)	$X^2 - 14Y^2 = 900(k^2 + 14k - 7)^2$	$[(464k+6720)x_{n+1} - (14k+210)y_{n+2},$ $(4k+56)y_{n+2} - (124k+1796)x_{n+1}]$
5)	$X^2 - 14Y^2 = 806404(k^2 + 14k - 7)^2$	$[(13904k+201376)x_{n+1} - (14k+210)y_{n+3},$ $(4k+56)y_{n+3} - (3716k+53820)x_{n+1}]$
6)	$X^2 - 14Y^2 = 196(k^2 + 14k - 7)^2$	$[(6503k+94185)x_{n+2} - (217k+3143)x_{n+3},$ $(58k+840)x_{n+3} - (1738k+25172)x_{n+2}]$
7)	$X^2 - 14Y^2 = 900(k^2 + 14k - 7)^2$	$[(16k+224)x_{n+2} - (434k+6286)y_{n+1},$ $(116k+1680)y_{n+1} - (4k+60)x_{n+2}]$

8)	$X^2 - 14Y^2 = 4(k^2+14k-7)^2$	$[(464k+6720)x_{n+2} - (434k+6286)y_{n+2},$ $(116k+1680)y_{n+2} - (124k+1796)x_{n+2}]$
9)	$X^2 - 14Y^2 = 900(k^2+14k-7)^2$	$[(13904k+201376)x_{n+2} - (434k+6286)y_{n+3},$ $(116k+1680)y_{n+3} - (3716k+53820)x_{n+2}]$
10)	$X^2 - 14Y^2 = 806404(k^2+14k-7)^2$	$[(16k+224)x_{n+3} - (13006k+188370)y_{n+1},$ $(3476k+50344)y_{n+1} - (4k+60)x_{n+3}]$
11)	$X^2 - 14Y^2 = 900(k^2+14k-7)^2$	$[(464k+6720)x_{n+3} - (13006k+188370)y_{n+2},$ $(3476k+50344)y_{n+2} - (124k+1796)x_{n+3}]$
12)	$X^2 - 14Y^2 = 4(k^2+14k-7)^2$	$[(13904k+201376)x_{n+3} - (13006k+188370)y_{n+3},$ $(3476k+50344)y_{n+3} - (3716k+53820)x_{n+3}]$
13)	$16X^2 - 14Y^2 = 64(k^2+14k-7)^2$	$[(k+14)y_{n+2} - (29k+420)y_{n+1},$ $(31k+449)y_{n+1} - (k+15)y_{n+2}]$
14)	$16X^2 - 14Y^2 = 57600(k^2+14k-7)^2$	$[(k+14)y_{n+3} - (869k+12586)y_{n+1},$ $(929k+13455)y_{n+1} - (k+15)y_{n+3}]$
15)	$16X^2 - 14Y^2 = 64(k^2+14k-7)^2$	$[(29k+420)y_{n+3} - (869k+12586)y_{n+2},$ $(929k+13455)y_{n+2} - (31k+449)y_{n+3}]$

Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the table 3 below:

Table 3: Parabolas

S.NO:	Parabola	(X,Y)
1)	$49(k^2+14k-7)X - 14Y^2 = 98(k^2+14k-7)^2$	$[(31k+449)x_{2n+2} - (k+15)x_{2n+3},$ $(2k+28)x_{n+2} - (58k+840)x_{n+1}]$
2)	$1680(k^2+14k-7)X - Y^2 = 100800(k^2+14k-7)^2$	$[(929k+13455)x_{2n+2} - (k+15)x_{2n+4},$ $(8k+12)x_{n+3} - (6952k+100688)x_{n+1}]$
3)	$(k^2+14k-7)X - 14Y^2 = 2(k^2+14k-7)^2$	$[(16k+224)x_{2n+2} - (14k+210)y_{2n+2},$ $(4k+56)y_{n+1} - (4k+60)x_{n+1}]$
4)	$15(k^2+14k-7)X - 14Y^2 = 450(k^2+14k-7)^2$	$[(464k+6720)x_{2n+2} - (14k+210)y_{2n+3},$ $(4k+56)y_{n+2} - (124k+1796)x_{n+1}]$
5)	$449(k^2+14k-7)X - 14Y^2 = 403202(k^2+14k-7)^2$	$[(13904k+201376)x_{2n+2} - (14k+210)y_{2n+4},$ $(4k+56)y_{n+3} - (3716k+53820)x_{n+1}]$

6)	$7(k^2+14k-7)X-14Y^2=98(k^2+14k-7)^2$	$[(6503k+94185)x_{2n+3}-(217k+3143)x_{2n+4},$ $(58k+840)x_{n+3}-(1738k+25172)x_{n+2}]$
7)	$15(k^2+14k-7)X-14Y^2=450(k^2+14k-7)^2$	$[(16k+224)x_{2n+3}-(434k+6286)y_{2n+2},$ $(116k+1680)y_{n+1}-(4k+60)x_{n+2}]$
8)	$(k^2+14k-7)X-14Y^2=2(k^2+14k-7)^2$	$[(464k+6720)x_{2n+3}-(434k+6286)y_{2n+3},$ $(116k+1680)y_{n+2}-(124k+1796)x_{n+2}]$
9)	$15(k^2+14k-7)X-14Y^2=450(k^2+14k-7)^2$	$[(13904k+201376)x_{2n+3}-(434k+6286)y_{2n+4},$ $(116k+1680)y_{n+3}-(3716k+53820)x_{n+2}]$
10)	$449(k^2+14k-7)X-14Y^2=403202(k^2+14k-7)^2$	$[(16k+224)x_{2n+4}-(13006k+188370)y_{2n+2},$ $(3476k+50344)y_{n+1}-(4k+60)x_{n+3}]$
11)	$15(k^2+14k-7)X-14Y^2=450(k^2+14k-7)^2$	$[(464k+6720)x_{2n+4}-(13006k+188370)y_{2n+3},$ $(3476k+50344)y_{n+2}-(124k+1796)x_{n+3}]$
12)	$(k^2+14k-7)X-14Y^2=2(k^2+14k-7)^2$	$[(13904k+201376)x_{2n+4}-(13006k+188370)y_{2n+4},$ $(3476k+50344)y_{n+3}-(3716k+53820)x_{n+3}]$
13)	$16(k^2+14k-7)X-14Y^2=32(k^2+14k-7)^2$	$[(k+14)y_{2n+3}-(29k+420)y_{2n+2},$ $(31k+449)y_{n+1}-(k+15)y_{n+2}]$
14)	$480(k^2+14k-7)X-14Y^2=28800(k^2+14k-7)^2$	$[(k+14)y_{2n+4}-(869k+12586)y_{2n+2},$ $(929k+13455)y_{n+1}-(k+15)y_{n+3}]$
15)	$16(k^2+14k-7)X-14Y^2=32(k^2+14k-7)^2$	$[(29k+420)y_{2n+4}-(869k+12586)y_{2n+3},$ $(929k+13455)y_{n+2}-(31k+449)y_{n+3}]$

Consider $p=x_{n+1}+y_{n+1}, q=x_{n+1}$. Note that $p>q>0$. Treat p,q as the generators of the Pythagorean triangle $T(X,Y,Z)$ where $X=2pq, Y=p^2-q^2, Z=p^2+q^2$.
Then the following results are obtained:

$$7X-4Y-3Z-(k^2+14k-7)=0.$$

$$\frac{2A}{P}=x_{n+1}y_{n+1}.$$

$3(Z-Y)$ is a nasty number.

$3(X-\frac{4A}{P})$ is a nasty number.

III. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the Diophantine equation, represented by hyperbola is given by $8x^2-7y^2=k^2+14k-7$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of equations and determine their integer solutions along with suitable properties.

REFERENCES

1. R.D. Carmichael, Theory of Numbers and Diophantine Analysis, Dover Publications, New York, 1950.
2. L.E. Dickson, History of theory of numbers, Vol.II, Chelsea Publishing co., New York, 1952.
3. L.J. Mordell, Diophantine Equations, Academic Press, London, 1969.
4. M.A. Gopalan and R. Anbuselvi, Integral solutions of $4ay^2-(a-1)x^2=3a+1$ Acta Ciencia Indica, XXXIV(1),2008,291-295.
5. M.A. Gopalan, etal, Integral points on the hyperbola $(a+2)x^2-ay^2=4a(k-1)+2k^2, a,k>0,$ Indian Journal of Science, 1(2),2012,125-126.
6. M.A. Gopalan, S. Devibala and R.Vidhyalakshmi, Integral points on the hyperbola $2X^2-3Y^2=5,$ American Journal of Applied Mathematics and Mathematical Sciences, 1(2012) 1-4.
7. S. Vidhyalakshmi,etal., Observations on the hyperbola $ax^2-(a+1)y^2=3a-1,$ Discovery,4(10), 2013,22-24.
8. K. Meena, M.A. Gopalan and S. Nandhini, On the binary quadratic Diophantine equation $y^2=68x^2+13,$ International Journal of Advanced Education and Research, 2,2017,59-63.
9. K. Meena, S. Vidhyalakshmi and R. Sobana Devi, On the binary quadratic equation $y^2=7x^2+32,$ International Journal of Advanced Science and Research, 2,2017,18-22.
10. K. Meena, M.A. Gopalan, S. Hemalatha, On the hyperbola $y^2=8x^2+16,$ National Journal of Multidisciplinary Research and Development, 2,2017,1-5.
11. M.A. Gopalan, K.K. Viswanathan and G. Ramya, On the positive Pell equation $y^2=12x^2+13,$ International Journal of Advanced Education and Research, 2, 2017,4-8.
12. K. Meena, M.A. Gopalan and V. Sivaranjani, On the positive Pell equation $y^2=102x^2+33,$ International Journal of Advanced Education and Research, 2(1),2017,91-96.
13. K. Meena, S. Vidhyalakshmi and N. Bhuvaneswari, On the binary quadratic Diophantine equation $y^2=10x^2+24,$ International Journal of Multidisciplinary Education and Research, 2,2017,34-39.
14. Gopalan,etal., Integral points on the hyperbola $(a+2)x^2-ay^2=4a(k-1)+2k^2, a,k>0,$ Indian Journal of Science, 1(2) ,2012,125-126.
15. M.A. Gopalan and V.Geetha, Observations on some special Pellian equations, Cayley J. Math, 2(2), 2013,109-118.
16. M.A. Gopalan, S. Vidhyalakshmi and A. Kavitha, On the integer solutions of binary quadratic equation, $x^2=4(k^2+1)y^2+4^t, k,t>0,$ BOMSR, 2,2014, 42-46.
17. K. Ambika, T.R. Usha rani, Observations on the Non-Homogeneous Binary Quadratic Diophantine Equation $5x^2-6y^2=5,$ Journal of Mathematics and Informatics, Vol-10,2017, 1-9.
18. S. Mallika, D. Hema, Observations on the hyperbola $8x^2-3y^2=20,$ International Journal on Research Innovations in Engineering Science and Technology(IJRIEST), Vol-3, Issue 4, April 2018, 575-582.